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ABSTRACT

We provide a necessary and sufficient condition on the equilibrium of a Walrasian economy for an increase in police expenditure to induce an increase in crime. This perverse effect is consistent with any appropriation technology and could arise even if the level of police protection is the socially optimal one.

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Keywords: Theft, Crime, Police, General Equilibrium, Laffer.

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1 Introduction

One of the more longstanding questions in the empirical literature on crime concerns the extent to which police affects crime.[1] While some papers have found a negative causal effect, it is safe to say that this literature has not yet provided a definitive answer. The main difficulty stems from the fact that while criminals may well react to an increase in police presence, police also tend to be deployed where there is crime.

Another characteristic of the empirical literature on crime and police is that it seems to stand on quite modest theoretical foundations. As Burdett, Lagos, and Wright (2003) aptly say, "much (although not all) work on the economics of crime uses partial equilibrium reasoning or empirical methods with very little grounding in economic theory." One of the possible reasons for this meager theoretical support is that models of crime are somewhat scarce. Indeed, Polinsky and Shavell (2000) and Chalfin and McCrary (2017), which are two recent surveys on law enforcement and deterrence, are centered exclusively on a simplified version of Becker's (1968) seminal model. Furthermore, most of the empirical papers that investigate the causal effect of police on crime do not explicitly formulate the structural model on which they are based, thereby making it difficult to make out the mechanism through which police may affect crime.

Although early economists were aware of the importance of theft as an allocation process, not until the 1960s was crime formalized as an economic activity performed by rational agents. Since Becker (1968), several strands of literature that adopt the economic approach to the study of crime have emerged. Early papers use models in which consumers and criminals react to incentives and meet in the proverbial

[1]Prominent exponents of this literature are Marvell and Moody (1996), Levitt (1997), Evans and Owens (2007), Di Tella and Schargrodsky (2004), Klick and Tabarrok (2005), Draca, Machin, and Witt (2011), Chalfin and McCrary (2018) and, more recently, Weisburd (2021), and Bindler and Hjalmarsson (2021). See also Levitt and Miles (2006) and Chalfin and McCrary (2017) for additional references.

market for offenses (see Ehrlich (1996) for an overview). Other papers adopt a search-theoretic approach to model an economy with theft, prominent examples being Burdett, Lagos, and Wright (2003, 2004). Finally, a few papers introduce theft into a Walrasian model, notably Usher (1987), Grossman (1994), and Dal Bó and Dal Bó (2011). A noteworthy attribute of the above models is that they predict that law enforcement unequivocally deters crime. The models that follow Becker's approach assume that the supply of criminal offenses is negatively related to the probability of apprehension. The search model proposed by Burdett, Lagos, and Wright (2003) exhibits multiple types of equilibria, all of which predict that police reduce crime. The same prediction arises from the model in Dal Bó and Dal Bó (2011). The reason is that in these models, while the relevant endogenous variables affect the level of crime, the level of crime does not have any feedback effect on the other variables.

One of the main ingredients of the economic approach to a theory of crime is its reliance on the role of incentives to determine individual behavior, criminal or otherwise. In a recent paper, Lasso de la Vega, Volij, and Weinschelbaum (2021) additionally require that the explanation fit the Walrasian model of an economy. They show that when theft is introduced in such a model in a way that allows all factors of production to be stealable, more police unequivocally reduce crime. But when only produced goods are subject to theft, they give an example in which an increase in police actually increases crime.

In this paper we show that such a perverse effect of police on crime is not due to unrealistic primitives of the economy that lead to a pathological example. Specifically we provide a necessary and sufficient condition on the equilibriun for the police to have a perverse effect on crime. Interestingly, this condition is essentially the condition for the Laffer curve to be downward sloping at a given ad valorem tax rate. The reason is the following. Theft imposes a tax on consumption goods whose rate we show to be negatively affected by the level of police. Since the associated tax revenue is the value of the stolen goods, which is precisely the level of crime, an increase in police protection will increase the level of crime whenever reducing the tax rate increases the tax revenue.

One may wonder if the above-mentioned condition is overly restrictive. We show that this is not the case by showing that, for any set of values of the endogenous variables, we can calibrate an economy that fits them and whose unique equilibrium satisfies the condition. One may object that the calibrated economy endows thieves with a very special appropriation technology. However, we also show that for any appropriation technology, one can build an economy with a Cobb-Douglas production function and CRRA preferences such that an increase in police protection induces an increase in crime. The mechanism is based on the fact that whereas, *coeteris paribus*, more police has a negative incentive on thieves, it also promotes economic activity which in turn makes theft more profitable. When the latter effect is strong enough, the perverse effect of police on crime is obtained.

One may also object that once a model allows for general equilibrium effects anything is possible. In particular, it may not be at all surprising that in such a model police increases crime. Our main result shows, however, that even accounting for feedback effects of crime on markets, whereas an increase in police spending may raise property crime, it unequivocally decreases the share of the GDP that is ultimately stolen. Hence, not anything goes.

One may still wonder whether the perverse effect of police on crime stems from a suboptimal level of police protection. For instance, Chalfin and McCrary (2018) suggest that "additional investments in police are unlikely to be socially beneficial unless police reduce violent crimes to at least a moderate degree." We show, however, that it may well be the case that even at the optimal level of police protection a perverse effect of police on crime still emerges. In particular, even though less police induces less crime, it is not socially worthwhile to reduce police spending. The reason is that one of the consequences of lower police protection is a decrease in output whose social cost may outweigh the benefits of lower crime and less police spending.

One final comment on our measure of crime. In this paper crime is measured by the aggregate amount of time devoted to theft. Since in our model crime is equivalent to theft, an alternative measure of crime would be the value of the stolen goods. It turns out that in equilibrium, these two measures are equivalent. This equivalence makes them the natural measure of crime.

The rest of the paper is organized as follows. Section 2 introduces the model and characterizes its competitive equilibrium. Section 3 establishes our main results. Finally, Section 4 concludes.

2 The model

We now present a version of the general equilibrium model with theft introduced by Lasso de la Vega, Volij, and Weinschelbaum (2021), under the assumption that only produced goods are subject to theft. The primitives of the model are the following. There is a publically available technology that transforms capital and labor into a consumption good, which will be henceforth referred to as *peanuts*. This technology is described by a constant returns to scale, monotone and concave production function $F(K, L)$. There is a continuum of individuals $I = [0, 1]$, characterized by an initial endowment of capital $\overline{k}_i$ and labor $\bar{l}_i$, and a quasilinear utility function $u_i(x, \ell) = \phi_i(x) + \ell$, that depends on the amount of peanuts, x, and leisure, ℓ, consumed. We assume that ϕ_i is strictly increasing, concave, and that $\lim_{x\to\infty} \phi_i'(x) = 0$. For notational convenience we will assume that all individuals are identical, namely $\phi_i = \phi$, $\overline{k}_i = \overline{K}$ and $\bar{l}_i = \overline{L}$ for all $i \in [0, 1]$. Furthermore, to avoid dealing with boundary problems, we assume that individuals can consume negative amounts of leisure and that the production function satisfy the Inada conditions. In particular, $\lim_{L\to 0} F_2(K, L) = \infty$.[2]

[2]For any function $f : \mathbb{R}^2 \to \mathbb{R}$, we denote by f_1 and f_2 its partial derivatives with respect to its first and second arguments. Also f_{jk}, for $j, k = 1, 2$, stand for the corresponding second derivatives.

There is an appropriation sector that uses labor to redistribute output from earners to thieves. Following Grossman (1994) and Dal Bó and Dal Bó (2011), we describe the appropriation technology by a function $A : \mathbb{R}^2_+ \to [0,1]$. The value $A(Y,T)$ is the proportion of the individuals income that gets stolen when the crime level is Y and police protection T. We call $A(Y,T)$ the *excise rate of theft* associated with Y and T. We assume that $A(0,T) = 0$, that A is increasing and strictly concave in its first argument, decreasing and strictly convex in its second argument, and that $A_{12} < 0$, namely the marginal excise rate of crime is decreasing in police protection. These assumptions imply that

$$A_1(Y,T) < \frac{A(Y,T)}{Y}$$

and that $\lim_{Y\to 0} A(Y,T)/Y = A_1(0,T)$. Namely, the marginal excise rate is lower than the average excise rate. We denote by $a(Y,T)$ the *average excise rate*, with the extension $a(0,T) = A_1(0,T)$. It is the proportion of wealth stolen per unit of time devoted to theft. It follows from our assumptions that $a(Y,T)$ is decreasing in both its arguments, and convex in its second argument. We summarize the data of the economy by $\mathcal{E} = \langle(\phi, \overline{K}, \overline{L},), F, A, T\rangle$.

Individuals, apart from consuming peanuts and leisure, devote some time to theft. A *bundle* for individual i is thus a triple $(x_i, \ell_i, y_i) \in \mathbb{R}_+ \times \mathbb{R} \times \mathbb{R}_+$ whose components are the amounts of peanuts, leisure, and time devoted to theft. We denote the set of bundles by $\mathcal{X}$.

When individual i devotes y_i units of his time to theft, the crime level is $Y = \int_0^1 y_i di$, and he gets a portion y_i/Y of the booty.[3] There is a level T of public police protection which is allocated uniformly across individuals and is financed by means of compulsory taxation.

An *allocation* in $\mathcal{E}$ consists of an input pair $(K,L) \in \mathbb{R}^2_+$, an assignment $(x,\ell,y) : [0,1] \to \mathcal{X}$ of bundles to individuals, and a crime level Y. An allocation is *feasible*

[3]For any real function f defined on $[0,1]$, we will sometimes write $\int f$ for $\int_0^1 f_i \, di$. All functions defined on $[0,1]$ are assumed to be integrable.

if

$$\begin{aligned}\int x &= F(K, L)\\ \overline{L} &= \int \ell + L + \int y + T\\ \overline{K} &= K\\ Y &= \int y.\end{aligned} \tag{1}$$

Namely, peanuts consumed are equal to peanuts produced, the sum of time devoted to leisure, labor, theft and police protection equals the total time available, capital used in the production process equals the amount of capital available, and the crime level is the per capita time devoted to theft.

2.1 Competitive equilibrium

We normalize the wage rate to be 1, and for simplicity, we assume that public police is financed by uniform taxation. The resources that an individual has available for the purchase of peanuts consist of the portion of his legitimate income (net of taxes) that is not stolen, plus the proceeds from his appropriation activities. Under our assumption that only produced goods are subject to theft, the returns to theft are given by $a(Y,T)pF(\overline{K}, L)$ and therefore, an individual's budget is given by

$$B = \left\{(x_i, \ell_i, y_i) : px_i \leq (1 - A(Y,T))(r\overline{K} + \overline{L} - \ell_i - y_i - T) + y_i a(Y,T)pF(\overline{K}, L)\right\}$$

The parameters that the individual takes as given are the price of peanuts p, the rental rate of capital r, the tax T, the crime level Y, and the returns to theft.[4] Note that $\overline{L} - \ell_i - y_i - T$ is the time that individual i devotes to labor. Also note that the relative price of peanuts (in terms of leisure) faced by the consumers is $p/(1 - A(Y,T))$. This is so because if a consumer wants to bring home one unit of peanuts he needs to buy $1/(1 - A(Y,T))$ units, since a proportion $A(Y,T)$ of them will be stolen.

[4] We ignore his share in the firms' profits since, given the constant returns to scale technology, profits will be 0 in equilibrium.

The concept of competitive equilibrium is the usual one.

Definition 1 A *competitive equilibrium* consists of a price of peanuts p^*, a rental rate of capital r^*, and a feasible allocation $\langle (L^*, K^*), (x^*, \ell^*, y^*), Y^* \rangle$, such that

1. The input pair (L^*, K^*) maximizes profits given p^* and r^*.
2. For all $i \in [0,1]$, the bundle (x_i^*, ℓ_i^*, y_i^*) maximizes the individual's utility given p^*, r^*, and Y^*.

2.2 Characterization of the equilibrium

Given our assumptions on preferences and technology any equilibrium allocation $\langle (L^*, K^*), (x^*, \ell^*, y^*), Y^* \rangle$ must satisfy $K^* > 0$, $L^* > 0$ and $x > 0$. Therefore, the necessary (and sufficient) conditions for profit maximization are:

$$\begin{aligned} p^* \frac{\partial F}{\partial L}(K^*, L^*) &= 1 \\ p^* \frac{\partial F}{\partial K}(K^*, L^*) &= r^* \end{aligned}$$

Namely, input prices must be equal to the value of their marginal productivity.

The first-order conditions for individual i's utility maximization are:

$$\phi'(x_i^*) = \frac{p^*}{1 - A(Y^*, T)} \tag{2}$$

$$1 - A(Y^*, T) \geq a(Y^*, T) p^* F(K^*, L^*) \quad \text{with equality if } y_i^* > 0 \tag{3}$$

$$p^* x_i^* = (1 - A(Y^*, T))(r^* K^* + L^*) + y_i^* a(Y^*, T) p F(K^*, L^*)$$

where Y^* is the crime level associated with the equilibrium allocation. Observe that in equilibrium $A(Y^*, T) < 1$, which follows from (2). Condition (3) is an arbitrage condition which states that the returns to theft cannot exceed the returns to labor, and that they must be equal if the individual devotes positive time to theft.

Finally, the allocation must satisfy the feasibility conditions (1).

Given that in equilibrium, the capital used by the firms, K^*, must be $\overline{K}$, it will be convenient to define the firm's short-run supply function. It is the function $Q : \mathbb{R}_{++} \to \mathbb{R}_+$ implicitly defined by

$$\begin{aligned} 1 &= p\frac{\partial F}{\partial L}(\overline{K}, L) \\ Q(p) &= F(\overline{K}, L) \end{aligned}$$

Similarly, it will be convenient to define the economy's aggregate demand function for peanuts. It is the function $X : \mathbb{R}_{++} \to \mathbb{R}_+$ implicitly defined by

$$\phi'(X(p)) = p.$$

Upon close observation of the above equilibrium conditions, and taking advantage of the definitions of the short-run aggregate supply and demand functions just defined, we can see that to find an equilibrium it is enough to solve

$$1 - A(Y,T) \geq a(Y,T)pQ(p) \quad \text{with equality if } Y > 0 \tag{4}$$

$$X\left(\frac{p}{1 - A(Y,T)}\right) = Q(p) \tag{5}$$

This is a system of two equations with two unknowns (p and Y). Once solved, the other variables are obtained by mere substitution. Indeed, the remaining variables, L^*, r^*, and ℓ^* are directly obtained from

$$\begin{aligned} F(\overline{K}, L^*) &= Q(p^*) \\ p^*\frac{\partial F}{\partial K}(\overline{K}, L^*) &= r^* \\ \overline{L} - T - Y^* &= \int \ell^*. \end{aligned}$$

Note that in equilibrium

$$Y^* = \frac{A(Y^*,T)}{1 - A(Y^*,T)}p^*Q(p^*). \tag{6}$$

Indeed, if $Y^* = 0$, this equality is trivially satisfied. And if $Y^* > 0$, it follows from (4). Recall that $p^*/(1 - A(Y^*,T))$ is the peanut price faced by the consumers. Therefore, the above equation says that in equilibrium, the aggregate time devoted to theft equals the value of the stolen goods at consumer prices. For that reason, it is natural to call Y^* the level of theft or of (property) crime. Also note that

$$Y^* = \frac{A(Y^*, T)}{1 - A(Y^*, T)} p^* Q(p^*) = A(Y^*, T) p^* Q(p^*) + A^2(Y^*, T) p^* Q(p^*) + \cdots$$

That is, property crime at the equilibrium crime level Y^* is not just the proportion $A(Y^*, T)$ of the GDP. The portion $A(Y^*, T)p^*Q(p^*)$ is only the peanuts stolen from the income legitimately earned by the agents. But property crime includes also the peanuts stolen from the stolen income as well.

Figure 1 depicts the equilibrium in the peanut market (where $A(Y^*, T)$ is denoted simply by A^*).

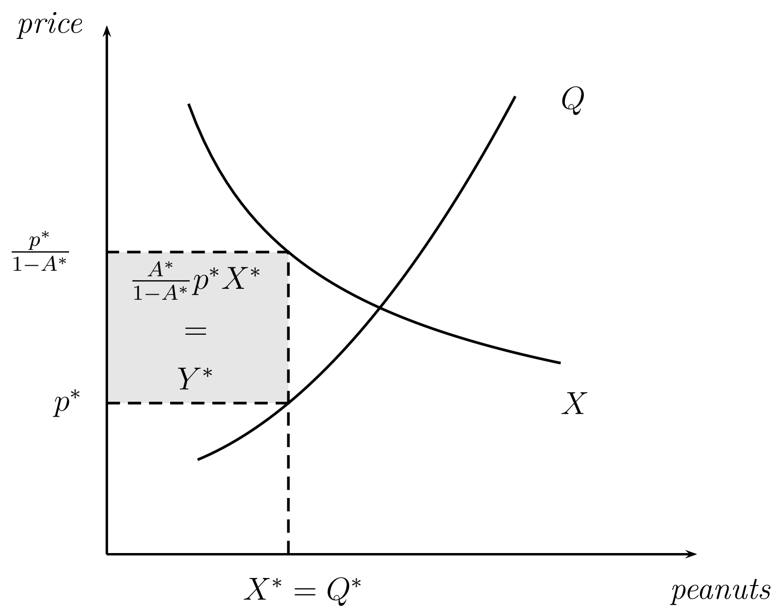

Figure 1: The peanut market.

The price faced by the consumers is $p^*/(1-A(Y^*, T))$ and the price faced by the firm is p^*. The difference is $p^* \frac{A(Y^*,T)}{1-A(Y^*,T)}$. As can be seen, theft has a similar effect to that of an ad valorem tax of $A(Y^*, T)/(1 - A(Y^*, T))$. It introduces a wedge between the effective price paid by the consumers and the one received by the firm. The difference is the value of the peanuts being stolen when one unit of peanuts is acquired. However, since by (6), the value of the stolen peanuts equals the value of the time spent on appropriation activities, this value ultimately dissipates.

Lasso de la Vega, Volij, and Weinschelbaum (2021) shows that there are economies with no equilibrium. However, it also shows that if the appropriation technology satisfies certain weak conditions, an equilibrium exists and is unique. This is stated in the following observation. Since it is short, we include a proof that fits this version of the model.

Observation 1 Let $\mathcal{E} = \langle(\phi, \overline{K}, \overline{L},), F, A, T\rangle$ be an economy. Then, if the appropriation technology A is bounded away from one, an equilibrium exists. If, furthermore, $\frac{a(Y,T)}{1-A(Y,T)}$ is non-increasing in Y, the equilibrium is unique.

Proof : See Appendix. □

For future reference, we define $g(Y,T) = \frac{a(Y,T)}{1-A(Y,T)}$. Given our assumptions on A, the function g satisfies $g_2 < 0$. Appropriation technologies A that are bounded away from one and for which $g(Y,T)$ is non-increasing in Y are said to be *regular*.

3 The effect of police on crime

We want to focus on the effect of police protection on the crime level. It can be seen from equations (4–5) that the peanut market affects the crime level and simultaneously the crime level affects the peanut market. For that reason, the effect of changes in police protection on crime may be ambiguous. If we observe Figure 1 we see that an increase in crime means that the area of the shaded rectangle grows. Therefore, we expect the perverse effect of police on crime to occur if certain conditions on the elasticites of demand and supply are met. The next theorem establishes these conditions.

Theorem 1 Consider an equilibrium of an economy in which police expenditure is T_0. If the equilibrium crime level Y^* is positive, and $g_1(Y^*, T_0) \leq 0$, then

1. an increase in police protection induces an increase in the equilibrium peanut price, $p(T_0)$, and output, $Q(p(T_0))$;

2. an increase in police protection induces a decrease in the equilibrium excise rate, $A(Y^*, T_0)$, and crime's implied tax rate of peanuts $A(Y^*, T_0)/(1 - A(Y^*, T_0))$;

3. an increase in police protection induces an increase in crime if and only if

$$1 > A(Y^*, T_0) > \frac{(\varepsilon - \eta)}{(1 + \eta)\,\varepsilon} \tag{7}$$

where η and ε are the elasticity of the supply function Q at p^* and of the demand function X at $p^*/\,(1 - A(Y^*, T_0))$, respectively.

Proof : Recall that $g(Y, T) = \frac{a(Y,T)}{1-A(Y,T)}$ and notice that any equilibrium of an economy $\mathcal{E} = \langle(\phi, \overline{K}, \overline{L},), F, A, T_0\rangle$ with positive crime level Y^* and peanut price p^* is characterized by

$$\begin{aligned} 1 &= g(Y^*, T_0)p^*Q(p^*) && (8) \\ X\big(\frac{p^*}{1 - A(Y^*, T_0)}\big) &= Q(p^*). && (9) \end{aligned}$$

These equations implicitly define the equilibrium crime level $Y(T)$ and peanut price $p(T)$ as functions of police protection in a neighborhood of T_0, with $Y(T_0) = Y^*$ and $p(T_0) = p^*$. By the implicit function theorem

$$\begin{aligned} p'(T_0) &= \frac{p^*QX'\,(A_2g_1 - A_1g_2)}{X'gA_1\,(p^*Q' + Q) + (1 - A)g_1Q\,((1 - A)Q' - X')} && (10) \\ Y'(T_0) &= -\frac{X'gA_2\,(p^*Q' + Q) + (1 - A)g_2Q\,((1 - A)Q' - X')}{X'gA_1\,(p^*Q' + Q) + (1 - A)g_1Q\,((1 - A)Q' - X')} && (11) \end{aligned}$$

where we have used the following simplifying notation: $g = g(Y^*, T_0)$, $g_1 = g_1(Y^*, T_0)$, $g_2 = g_2(Y^*, T_0)$, $A = A(Y^*, T_0)$, $A_1 = A_1(Y^*, T_0)$, $A_2 = A_2(Y^*, T_0)$, $Q = Q(p^*)$, $Q' = Q'(p^*)$, and $X' = X'(\frac{p^*}{1-A(Y^*,T_0)})$. Recall that in equilibrium $A(Y^*, T_0) < 1$ and hence these values are well defined.

Given our assumptions on the appropriation technology and that $g_1(Y^*, T_0) \leq 0$, we have that the denominator of the above expressions is negative and the numerator of (10) is negative as well. Therefore, $p'(T) > 0$. As a result, since $Q'(p) > 0$, we also obtain that the equilibrium level of output is increasing in T. This proves the first part of the theorem.

Also, we have

$$\begin{aligned} \frac{dA}{dT} &= A_1 Y'(T) + A_2 \\ &= \frac{(1-A)Q\left(A_2 g_1 - A_1 g_2\right)\left((1-A)Q' - X'\right)}{X' g A_1 \left(p^* Q' + Q\right) + (1-A) g_1 Q \left((1-A)Q' - X'\right)} \end{aligned}$$

which, given the properties of g and A, can be checked to be negative. This implies that the ratio $A/(1-A)$ is also decreasing in T, which proves the second part of the theorem.

Since the denominator of (11) is negative,

$$\begin{aligned} Y'(T_0) > 0 \Leftrightarrow \quad & -X' g A_2 \left(p^* Q' + Q\right) && < \quad (1-A) g_2 Q \left((1-A)Q' - X'\right) \\ \Leftrightarrow \quad & -X' \frac{A}{Y^*(1-A)} A_2 \left(p^* Q' + Q\right) && < \quad (1-A) \frac{A_2}{Y^*(1-A)^2} Q \left((1-A)Q' - X'\right) \\ \Leftrightarrow \quad & -X' A A_2 \left(p^* Q' + Q\right) && < \quad A_2 Q \left((1-A)Q' - X'\right) \\ \Leftrightarrow \quad & -X' A \left(p^* Q' + Q\right) && > \quad Q \left((1-A)Q' - X'\right) \end{aligned}$$

where we have used the fact that $g_2 = A_2/(Y^*(1-A)^2)$ and that $A_2 < 0$. Dividing both sides by Q, and denoting by η the elasticity of the supply function at p^*, we obtain that $Y'(T_0) > 0$ if and only if

$$-X' A \left(1 + \eta\right) > (1-A)Q' - X'.$$

Multiplying both sides by $p^*/\left(Q(1-A)\right)$, and taking into account that in equilibrium $Q = X$, we obtain

$$-\frac{X'}{X} \frac{p}{(1-A)} A \left(1 + \eta\right) > p \frac{Q'}{Q} - \frac{X'}{X} \frac{p}{(1-A)}.$$

Denoting by ε the elasticity of the demand function X at $p^*/\left(1 - A(Y^*, T_0)\right)$, the above inequality can be written as

$$-\varepsilon A \left(1 + \eta\right) > \left(\eta - \varepsilon\right)$$

which holds if and only if

$$1 > A > \frac{(\varepsilon - \eta)}{(1+\eta)\,\varepsilon}$$

which proves the third statement of the theorem. □

The third part of Theorem 1 says that the effect of police on crime is ambiguous; under certain conditions more police induces more crime and under other conditions reduces crime. It could be argued that this is not surprising since once general equilibrium effects are allowed, anything can happen. However, as the first and second parts of the theorem show, even with feedback effects an increase in police expenditure unambiguously increases the price and quantity of peanuts and reduces the equilibrium excise rate. Therefore, the above ambiguity is not a forgone conclusion.

Theorem 1 states that a necessary and sufficient condition for the police to have an adverse effect on crime is that the equilibrium excise rate be bigger than $\frac{(\varepsilon-\eta)}{(1+\eta)\varepsilon}$. It can be checked that this inequality implies that $\varepsilon < -1$. This means that a sufficient condition for the police to reduce crime is that the demand for peanuts be inelastic. Condition (7) can be equivalently written as

$$0 < \frac{\varepsilon - \eta}{\eta(1+\varepsilon)} < \frac{A(Y^*, T_0)}{1 - A(Y^*, T_0)} \tag{12}$$

Recall that $t^* = \frac{A(Y^*,T_0)}{1-A(Y^*,T_0)}$ is the ad valorem "tax rate" imposed by crime on peanuts. Condition (12) is precisely the condition for the Laffer curve to be downward sloping at the ad valorem tax rate of t^*. The intuition for this equivalence is the following. As the second part of Theorem 1 shows, an increase in police expenditure unambiguously results in a reduction in crime's tax rate, t^*, on peanuts. Therefore, since the level of crime is precisely the tax revenue associated with t^*, an increase in police expenditure will increase crime if and only if the Laffer curve is negatively sloped at the equilibrium. The only difference between the condition established in Theorem 1 and the condition for the Laffer curve to be downward sloping, is that whereas an ad valorem tax rate is an exogenous variable determined by the government, the tax rate imposed by crime, t^*, is endogenously determined in equilibrium.

As a corollary of Theorem 1 we obtain that by choosing an appropriate appropriation technology, we can calibrate an economy so that its equilibrium variables take any given values and where police has an adverse effect on crime. This is stated in the following proposition.

Proposition 1 Let T_0 be a given level of public police protection. Let $p^* > 0$, $Q^* > 0$, and $Y^* > 0$, be a price, quantity of peanuts, and a crime level. There exists an economy such that at its unique equilibrium the price, output and crime level are given by p^*, Q^*, and Y^*, respectively, and such that a small increase in police protection results in an increase in crime.

Proof : We will build an economy $\mathcal{E} = \langle(\phi, \overline{K}, \overline{L}), F, A, T_0\rangle$ with peanut price p^*, output Q^*, equilibrium crime level $Y^* > 0$, and such that if police protection is slightly increased the crime level will also increase. Theorem 1 establishes a condition for this to occur. Our task is then to build an economy that satisfies it.

If p^*, Q^*, and Y^* are the equilibrium price, output, and crime level of some economy with police protection given by T_0, using equation (6) we obtain that the equilibrium excise rate of theft is given by,

$$A(Y^*, T_0) = \frac{Y^*}{p^*Q^* + Y^*}.$$

Let $0 < \alpha < A(Y^*, T_0)$. This α can be found since $Y^* > 0$. The production function of the economy is chosen to be $F(K, L) = K^\alpha L^{1-\alpha}$. Therefore, when the capital level is fixed at K, the corresponding short-run supply function is $Q(p) = K((1-\alpha)p)^{\frac{1-\alpha}{\alpha}}$. Note that the elasticity of supply is $\eta = \frac{1-\alpha}{\alpha}$. We now choose the capital endowment to be $\overline{K}$ such that $Q(p^*) = Q^*$. The choice of $\overline{L}$ is arbitrary although it can be chosen to be large enough so that the resulting equilibrium leisure is positive.

We now choose the utility function. Let $\hat{\varepsilon} < -1$ and $\hat{c} > 0$ be the unique solution of

$$\frac{(\varepsilon/2 - \eta)}{(1+\eta)\,\varepsilon/2} = A \tag{13}$$

$$\left(\frac{p^*}{c(1 - A(Y^*, T_0))}\right)^{\varepsilon} = Q^* \tag{14}$$

By the intermediate value theorem, this $\hat{\varepsilon}$ can be found because the left-hand side of (13), as a function of ε, is continuous in $(-\infty, 0)$, equals 1 when $\varepsilon = -2$, and

$$\lim_{\varepsilon \to -\infty} \frac{(\varepsilon/2 - \eta)}{(1+\eta)\,\varepsilon/2} = 1/(1+\eta) = \alpha < A(Y^*, T_0).$$

Note that since the left-hand side of (13) is increasing in ε, we have that

$$\frac{(\hat{\varepsilon} - \eta)}{(1+\eta)\,\hat{\varepsilon}} < A. \tag{15}$$

Once $\hat{\varepsilon}$ is found, $\hat{c}$ is obtained by solving equation (14).

We choose the consumers' utility function to be

$$\phi(x) = \hat{c}\,\frac{x^{1+1/\hat{\varepsilon}}}{1 + 1/\hat{\varepsilon}}.$$

Consequently, the corresponding demand function is $X(p) = (p/\hat{c})^{\hat{\varepsilon}}$, whose elasticity is constant and equal to $\hat{\varepsilon}$.

It remains to choose the appropriation technology. It will be given by

$$A(Y, T) = \frac{dY}{Y + p^* Q^* (\frac{T}{1+T} + e)}$$

where $e = \frac{1}{2(1+T_0)}$ and $d = \frac{\frac{p^*Q^*}{2} + p^*Q^*T_0 + T_0Y^* + Y^*}{p^*Q^* + p^*Q^*T_0 + T_0Y^* + Y^*}$. It is routine to check that $1 = g(Y^*, T_0)p^*Q^*$. Also, by the choice of the utility function (see equation (14)), $X(\frac{p^*}{1-A(Y^*,T_0)}) = Q^*$. This means that the economy $\mathcal{E} = \langle(\phi, \overline{K}, \overline{L}), F, A, T_0\rangle$ has an equilibrium with price, quantity and crime level given by p^*, Q^*, and Y^*. Furthermore, since $d < 1$, $A(Y, T)$ is bounded away from one. Also, it can be checked that $g_1(Y, T) < 0$. Hence, the appropriation technology is regular and, by Observation 1, the equilibrium is unique. By construction, inequality (15) holds which, implies that inequality (7) holds. By Theorem 1, we conclude that an increase in police protection increases the crime level. $\square$

One may argue that the perverse effect of police on crime identified in the above theorem results from a very peculiar appropriation technology. The next proposition, however, shows that this kind of perverse effect is compatible with any appropriation technology that is bounded away from one.

Proposition 2 Let T_0 be a given level of public police protection. Let A be an appropriation technology that is bounded away from one. Then there is an economy with appropriation technology A and police protection T_0 such that in equilibrium a small increase in police results in an increase in crime.

Proof : We will build an economy $\mathcal{E} = \langle(\phi, \overline{K}, \overline{L}), F, A, T_0\rangle$ with a positive equilibrium crime level $Y^* > 0$ and equilibrium price p^*, such that if police protection is slightly increased the crime level will also increase. Theorem 1 establishes a condition for this to occur. Our task is then to build an economy that satisfies it.

Let $\alpha \in (0,1)$ such that $A(Y, T_0) > \alpha$ for some Y, let $F(K, L) = K^\alpha L^{1-\alpha}$, and let $\overline{K} = 1$. As a result, the associated short-run aggregate supply function is given by $Q(p) = \big((1-\alpha)p\big)^{\frac{1-\alpha}{\alpha}}$ whose elasticity is $\eta = (1-\alpha)/\alpha$.

Since A is bounded away from one, we have that $\lim_{Y\to\infty} g(Y, T) = 0$. Therefore, for any Y, there is $\hat{Y} \geq Y$ such that $g_1(\hat{Y}, T_0) < 0$. Consequently, we can choose $Y^* > 0$ such that both $A(Y^*, T_0) > \alpha$ and $g_1(Y^*, T_0) < 0$. This Y^* will be the equilibrium crime level in the economy we are looking for.

For any $b > 0$, consider the utility function given by $\phi_b(x) = b\,ln(x)$. The associated demand function is $X_b(p) = b/p$. The peanut market clearing condition $X_b(\frac{p}{1-A(Y^*,T_0)}) = Q(p)$ can be written as

$$\frac{b}{p}\,(1 - A(Y^*, T_0)) = Q(p)$$

It can be checked that this equation has a unique solution, which we denote by $p(b)$. Note that $\partial p/\partial b > 0$ and that $\lim_{b\to\infty} p(b) = \infty$.

Consider now the function

$$f(b) = g(Y^*, T_0)p(b)Q(p(b))$$

We have that f is increasing in b, $f(0) = 0$ and $\lim_{b\to\infty} f(b) = \infty$. By the intermediate value theorem, there is $\hat{b}$ such that $f(\hat{b}) = 1$. This means that $p^* = p(\hat{b})$ and Y^* satisfy

$$\begin{aligned} 1 &= g(Y^*, T_0)p^*Q(p^*) && (16) \\ X_{\hat{b}}(\frac{p^*}{1 - A(Y^*, T_0)}) &= Q(p^*) && (17) \end{aligned}$$

In other words, p^* and Y^* are an equilibrium price and a crime level of the economy $\mathcal{E}_{\hat{b}} = \left\langle(\phi_{\hat{b}}, \overline{K}, \overline{L}), F, A, T_0\right\rangle$, where $\overline{L}$ is arbitrary but can be chosen so that the equilibrium per capita leisure is positive.

We have built an auxiliary economy with an equilibrium price p^* and crime level Y^*. However, this equilibrium does not necessarily satisfy the condition of Theorem 1. We now build a collection of economies with the same equilibrium price and crime level and show that one of them satisfies this condition.

For any $\varepsilon < -1$, let $b(\varepsilon)$ be implicitly defined by

$$\left(\frac{p^*}{b(\varepsilon)}\right)^{\varepsilon} = Q(p^*)$$

namely,

$$b(\varepsilon) = Q(p^*)^{-\frac{1}{\varepsilon}}p^*$$

and define the following utility function: $\phi^\varepsilon(x) = b(\varepsilon)\frac{x^{1+1/\varepsilon}}{1+1/\varepsilon}$. Note that the corresponding demand function is $X^\varepsilon(p) = \left(\frac{p^*}{b(\varepsilon)}\right)^{\varepsilon}$. Let $\mathcal{E}^\varepsilon = \langle(\phi^\varepsilon, \overline{K}, \overline{L}), F, A, T_0\rangle$ be the economy that is obtained from $\mathcal{E}_{\hat{b}}$ by replacing the utility function $\phi_{\hat{b}}$ by ϕ^ε. It can be checked that p^* and Y^* are equilibrium values of both the economies $\mathcal{E}^\varepsilon$ and $\mathcal{E}_{\hat{b}}$. Indeed, if we substitute p^* for p and Y^* for Y in $\mathcal{E}^\varepsilon$'s equilibrium conditions

$$\begin{aligned} 1 &= g(Y, T_0)pQ(p) && (18) \\ \left(\frac{p^*}{(1 - A(Y, T_0))c(\varepsilon)}\right)^{\varepsilon} &= Q(p^*) && (19) \end{aligned}$$

and compare them with equations (16–17) we see that these conditions hold. See Figure 2.

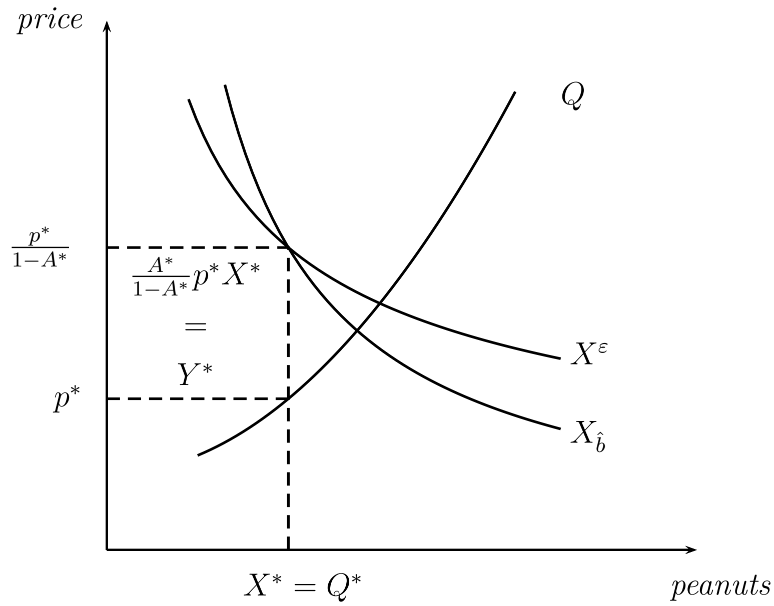

Figure 2: The pivoted demand function.

We now single out the economy $\mathcal{E}$ alluded to in the statement of the proposition. Since p^* does not depend on ε and since $\eta = \frac{1-\alpha}{\alpha}$,

$$\lim_{\varepsilon \to -\infty} \frac{(\varepsilon - \eta)}{(1+\eta)\,\varepsilon} = \frac{1}{1+\eta} = \alpha < A(Y^*, T_0).$$

Therefore, we can find $\tilde{\varepsilon}$ sufficiently negative so that

$$\frac{(\tilde{\varepsilon} - \eta)}{(1+\eta)\,\tilde{\varepsilon}} < A(Y^*, T_0).$$

Given that $g_1(Y^*, T_0) < 0$, this shows that the economy $\mathcal{E}^{\tilde{\varepsilon}}$ satisfies Theorem's 1 conditions. Therefore, $Y'(T_0) > 0$ and $\mathcal{E}^{\tilde{\varepsilon}}$ is the economy $\mathcal{E}$ that we were looking for. $\square$

3.1 Optimal police protection

In this section we show that even when police protection is set at the optimal level, it may well be the case that more police induces more crime.

Let $\mathcal{E} = \left\langle (\phi, \overline{K}, \overline{L}), F, A, T \right\rangle$ be an economy with a regular appropriation technology, and let $Y(T)$ the equilibrium crime level, which is assumed to be positive, and let $p(T)$ be the equilibrium price. Denote by $X^*(T)$ and $Q^*(T)$ the equilibrium quantity of peanuts demanded and produced. Namely, $X^*(T) = X(p(T))$, and $Q^*(T) = Q(p(T))$. Given that preferences are quasilinear, we can evaluate the social desirability of allocations by the associated utility they generate. Thus, the social welfare corresponding to the equilibrium allocation when police protection is T, is given by

$$W(T) = \phi(X^*(T)) + \overline{L} - c(Q^*(T)) - Y(T) - T$$

where $c : \mathbb{R}_+ \to \mathbb{R}_+$ is the short-run cost function associated with the production function F. Namely, c is implicitly defined by $Q = F(\overline{K}, c(Q))$.[5] The optimal level of public police protection satisfies

$$\phi'(X^*(T))X^{*\prime}(T) = c'(Q^*(T))Q^{*\prime}(T) + Y'(T) + 1.$$

In other words, it equalizes the marginal cost of police with its marginal benefit. The marginal cost consists of three components: the production cost of the additional output induced by the additional police, the increase (which may be negative) in the crime level, and the additional expenditure on police. The marginal benefit is the increase in the consumers' utility due to the additional consumption of peanuts.

Since in equilibrium $X^*(T) = Q^*(T)$, $\phi'(X^*(T)) = p(T)/(1 - A(Y(T), T))$ and $c'(Q^*(T)) = p^*(T)$, we have that the optimal level of public police protection satisfies

$$\frac{A(Y(T), T)}{1 - A(Y(T), T)} p(T)Q^{*\prime}(T) = 1 + Y'(T). \tag{20}$$

[5] Since the wage rate is 1, $c(Q)$ is the minimum amount of labor required to produce Q.

By part 1 of Theorem 1, we have that the left-hand side of this equation is positive.[6] Therefore, it may well be the case that even at the optimal level of police protection we have that $Y'(T) > 0$. The following example illustrates such an instance.

Example 1 Consider the economy $\mathcal{E} = \left\langle (\phi, \overline{K}, \overline{L}), F, A, T \right\rangle$ where $\phi(x) = x(9 - x/2)$, $\overline{K} = 1$, $F(K, L) = \sqrt{KL}$, and $A(Y, T) = \frac{3}{4} \frac{Y}{1+Y+2\frac{T}{1+T}}$. With these data, the aggregate demand function is given by $X(p) = 9 - p$ and the short-run supply function is given by $Q(p) = p/2$. The unique equilibrium of this economy results from the solution of equations (4)–(5), which is

$$Y(T) = \frac{5-3T}{3T+3} + \sqrt{\frac{57T+25}{T+1}} \qquad p(T) = 1 + \frac{1}{3}\sqrt{\frac{57T+25}{T+1}}.$$

The equilibrium crime rate $Y(T)$ is plotted in Figure 3. It can be seen that for $T < 11/21$, the crime rate increases with T. In particular, the maximum crime rate is not attained by reducing police funding to 0. Figure 3 also depicts the equilibrium social welfare as a function of police protection. It can be seen that it attains its maximum at $T^* = 0.419 < 11/21$. Therefore, in this economy, even at the optimal level of public police protection, an increase in police induces an increase in crime.

One would wonder why the social planner would not want to reduce police protection, even when at the optimum such a reduction would induce a decrease in crime. The answer can be seen in equation (20): a reduction in police protection decreases the equilibrium output with a corresponding reduction in consumer surplus, which turns out not to be compensated by the savings in police expenditure and the reduction in crime.

[6] If there were no feedback effect of crime, $Q^{*\prime}$ would be zero and at the optimal level of police we would have that $Y'(T) = -1$.

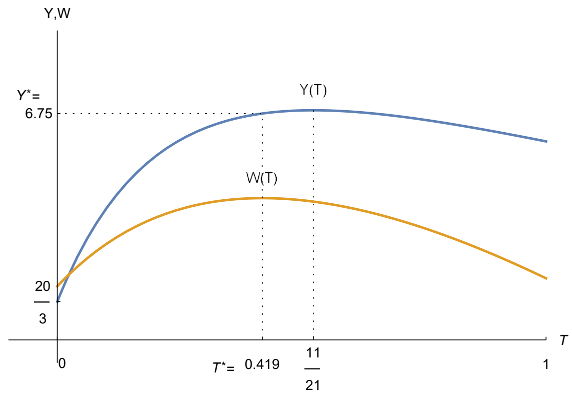

Figure 3: The equilibrium crime rate as a function of police protection and the optimal level of police protection.

4 Concluding remarks

There is a vast empirical literature whose aim is to measure the causal effect of police on crime. It is safe to say that, thus far, the data have not convincingly showed the negative relationship predicted by most models. In this paper we showed that an ambiguous relationship between police and crime is consistent with a textbook general equilibrium model.

The economic theory of crime postulates that criminals are rational agents who respond to incentives. Thus, one of its main components is a supply of criminal activity that falls as the opportunity cost of crime rises. However, this supply function is not the only component of a general equilibrium theory. As a result, although *coeteris paribus* an exogenous increase in police shifts the supply of criminal offenses downwards, it may well be the case that the equilibrium level of crime goes up once all the general equilibrium effects are considered. Whether or not this is theoretically possible depends on the details of the whole model and not only on the supply function. In fact, with the exception of Lasso de la Vega, Volij, and Weinschelbaum (2021), most of the existing general equilibrium models of crime predict that increases in police reduce crime. The mechanism at work in their model is based on the fact that although, *coeteris paribus*, more police reduce the incentives to engage in theft, it also induces economic prosperity which in turn increases the returns to theft. In this paper we have provided a necessary and sufficient condition for this mechanism to work. We have further demonstrated that for this mechanism to work it is not necessary to postulate any specific appropriation technology. On the contrary, for any appropriation technology simple economies can be built in which the equilibrium effect of an exogenous increase in police spending actually results in higher levels of crime. Furthermore, this perverse effect can take place even at the socially optimal level.

A Appendix

Proof of Observation 1 By our assumptions on ϕ and F, for any fixed Y, equation (5) has a unique solution, which we denote by $p(Y)$. It can be checked that $p(Y)$ is non-increasing in Y and, consequently, $Q(p(Y))$ is non-increasing in Y as well. Therefore, $\mathcal{E}$ has an equilibrium if

$$1 - A(Y,T) \geq a(Y,T)p(Y)Q(p(Y)) \quad \text{with equality if } Y > 0.$$

Since A is bounded away from one, this is equivalent to

$$1 \geq \frac{a(Y,T)}{1 - A(Y,T)} p(Y)Q(p(Y)) \quad \text{with equality if } Y > 0. \tag{21}$$

If $1 \geq \frac{a(0,T)}{1-A(0,T)}p(0)Q(p(0))$, then $Y^* = 0$ solves (21). If, on the other hand, $1 < \frac{a(0,T)}{1-A(0,T)}p(0)Q(p(0))$, then, given that $p(Y)Q(p(Y))$ is non-increasing, that A is bounded away from one, and that $a(Y,T)$ goes to 0 as Y goes to ∞, we have that $\frac{a(Y,T)}{1-A(Y,T)}p(Y)Q(p(Y)) \to 0$ as Y goes to ∞. By the intermediate value theorem, there is Y^* such that $1 = \frac{a(Y^*,T)}{1-A(Y^*,T)}p(Y^*)Q(p(Y^*))$ and an equilibrium exists. If $\frac{a(Y,T)}{1-A(Y,T)}$ is non-increasing, this Y^* is unique and so is the equilibrium. $\square$

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